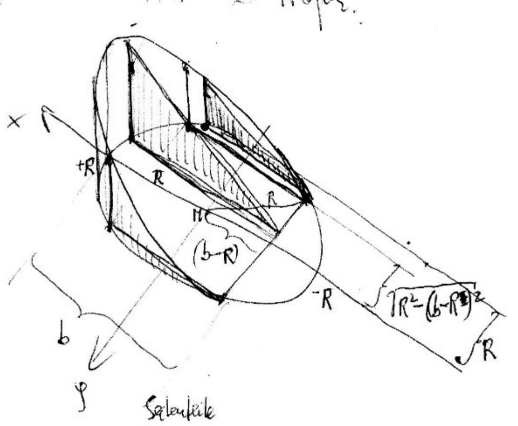
Dreieck! Trapez!



Seite₁ · Seite₂ h = z

$$A_{\text{Dreieck}} = \frac{1}{2}\cdot\left[\sqrt{R^2-\varphi^2}+b-R\right]\cdot\frac{h}{b}\left(\sqrt{R^2-\varphi^2}-R+b\right)$$

Bez.

$$V_{\text{vier Dreiecke}} = \cancel{2}\cdot\int_{\varphi=0}^{\sqrt{R^2-(b-R)^2}} \frac{1}{\cancel{2}}\cdot\frac{h}{b}\left(\sqrt{R^2-\varphi^2}+b-R\right)^2$$

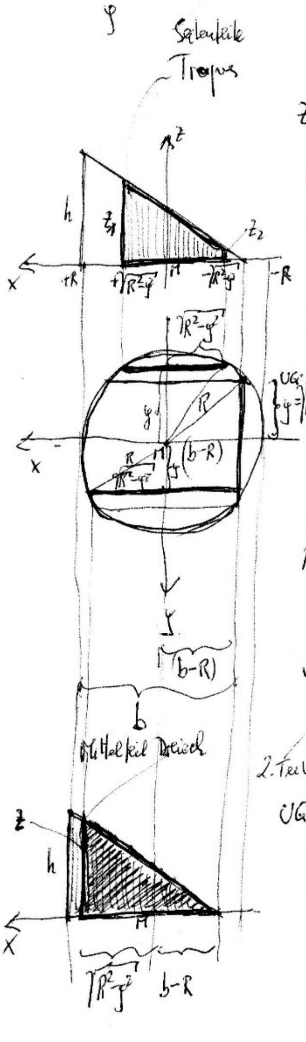
2 Teile!

$$V_{\text{vier Dreiecke}} = \int_{\varphi=0}^{\sqrt{1-(1{,}5-1)^2}=\sqrt{3}/2} \frac{1}{1{,}5}\left(\sqrt{1-\varphi^2}+1{,}5-1\right)d\varphi = \int_{\varphi=0}^{\sqrt{3}/2}\frac{2}{3}\left(\sqrt{1-\varphi^2}+1/2\right)d\varphi =$$

$$= \frac{4\cdot\sqrt{3}\,\pi+45}{4\cdot 9\cdot\sqrt{3}} \approx 1{,}07075\ldots$$

$$\underline{\underline{V_T+V_D}} = 0{,}0603\ldots + 1{,}0707 = \underline{\underline{1{,}131144\ldots}}$$

OK!



$$z = \frac{h}{b}\left(x-R+b\right)$$

$$z_1 = \frac{h}{b}\left(\sqrt{R^2-\varphi^2}-R+b\right)$$

$$z_2 = \frac{h}{b}\left(-\sqrt{R^2-\varphi^2}-R+b\right)$$

$$\underline{\underline{A_{\text{Trapez}}}} = \left(z_1+z_2\right)\cdot 2\cdot\sqrt{R^2-\varphi^2}/2 \quad (\text{Summe d. Parallelseiten} \times \text{Höhe}/2)$$

$$= \frac{h}{b}\left(\sqrt{\sim}-R+b-\sqrt{\sim}-R+b\right)\cdot\sqrt{R^2-\varphi^2}$$

$$= \underline{2\cdot\frac{h}{b}\left(-R+b\right)\sqrt{R^2-\varphi^2}}$$

$h=1$

$b=1{,}5$

$R=1$

$$A_{\text{Trapez}}(\varphi) = 2\cdot\frac{1}{\frac{3}{2}}\left(-1+1{,}5\right)\cdot\sqrt{1-\varphi^2}$$

$$= \frac{4}{3}\cdot 0{,}5\cdot\sqrt{1-\varphi^2} = \frac{2}{3}\sqrt{1-\varphi^2}$$

$$V_{\text{vier Trapeze}} = 2\cdot\int_{\varphi=\sqrt{1-(1{,}5-1)^2}=\sqrt{3}/2}^{1}\frac{2}{3}\cdot\sqrt{1-\varphi^2}\,d\varphi = \frac{2\cdot\sqrt{3}\cdot\pi-9}{2\cdot 9\cdot\sqrt{3}} \approx 0{,}06039\ldots$$

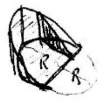
$$\underbrace{(1{,}5-1)^2}_{0{,}5^2 = 0{,}25}\qquad 0{,}75 = 3/4$$

2 Teile!

$$\text{UG: } \varphi = \sqrt{R^2-(b-R)^2}$$

Volumen der beiden „Seitenteile" „Wangen": „$\int A_{\text{Trapez}}\cdot d\varphi$"

Grenzfälle: 1) Trapez: $b = R (= 1)$ $\Rightarrow$ Zylinderhuf



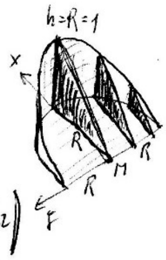
Trapez „verschwindet"
nur mehr Dreiecke!

$$A_{Trapez} = 2\,\frac{h}{b}\left(-R+b\right)\cdot\sqrt{R^2-y^2}$$

$h = 1$ (wie immer!) $\quad b = R$

$$A_{Trapez} = 2\cdot\frac{1}{b}\underbrace{\left(-R+R\right)}_{\emptyset}\cdot\sqrt{R^2-y^2} = \emptyset$$

2) Dreieck: $b = R (=1)$



b=R

$$A_{Dreieck} = \frac{1}{2}\,\frac{h}{b}\left(\sqrt{R^2-y^2}-R+b\right)^2$$

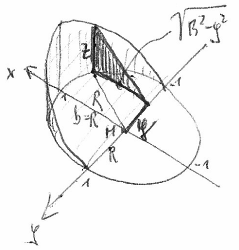
$h = 1$ (wie immer!)

$$A_{Dreieck} = \frac{1}{2}\cdot\frac{1}{b}\Big(\sqrt{R^2-y^2}\underbrace{-R+R}_{\emptyset}\Big)^2 = \frac{1}{2b}\cdot\left(R^2-y^2\right)$$

$$V_D = \cancel{2}\int_{y=0}^{\sqrt{R^2-(b-R)^2}=\sqrt{1-(1-1)^2}=1} \cancel{\tfrac{1}{2}}\cdot\frac{1}{1}\left(1-y^2\right)dy = \int_{y=0}^{1}\left(1-y^2\right)dy = \left[y-\frac{y^3}{3}\right]_0^1 = \overset{OG}{1-\frac{1}{3}} - \overset{UG}{\emptyset} = 2/3\,!$$

Vol.-Integration über Dreiecke alleine(!) funktioniert nur für $0 < b \leq R$.

Ist $b > R$ (genaugenommen $R < b < 2R$) ergeben sich dreieck- und trapezförmige Schnittflächen!

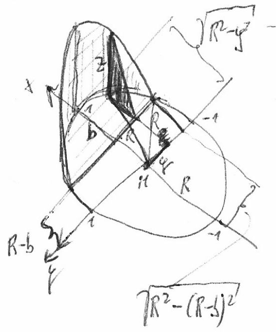


$$z = \frac{h}{b}(x - R + b)$$

$$\left.\begin{matrix} h=1 \\ R=1 \\ b=1 \end{matrix}\right\} \quad z = \frac{1}{1}(x-1+1) = x = \sqrt{R^2-y^2} = \sqrt{1-y^2}$$

$$A_{\triangle} = \frac{1}{2}\sqrt{R^2-y^2}\cdot\sqrt{R^2-y^2} = \frac{1}{2}\left(R^2-y^2\right) = \frac{1}{2}\left(1-y^2\right)$$

$$V = \cancel{2}\cdot\int_0^1 \cancel{\tfrac{1}{2}}(1-y^2)\,dy = \left[y - \frac{y^3}{3}\right]_0^1 = \overset{OG}{1 - \frac{1^3}{3}} - \overset{UG}{\varnothing} = \underline{\underline{2/3}} \checkmark$$



$$z = \frac{h}{b}(x - R + b) = \frac{h}{b}\left(\sqrt{R^2-y^2} - R + b\right)$$

$$\underline{0 < b \leq R} \;!$$

$$\sqrt{R^2-y^2} - (R-b) = \sqrt{R^2-y^2} - R + b$$

$$A_{\triangle} = \frac{1}{2}\left(\sqrt{R^2-y^2} - R + b\right)\cdot\frac{h}{b}\left(\sqrt{R^2-y^2} - R + b\right) = \frac{1}{2}\frac{h}{b}\left(\sqrt{R^2-y^2} - R + b\right)^2$$

$h = 1$ (wie immer!)

$R = 1$

$$A_{\triangle} = \frac{1}{2}\cdot\frac{1}{b}\left(\sqrt{1-y^2} - 1 + b\right)^2$$

$$V = \cancel{2}\cdot\int_{y=0}^{\sqrt{R^2-(R-b)^2} = \sqrt{1-(1-b)^2}} \cancel{\tfrac{1}{2}}\cdot\frac{1}{b}\cdot\left(\sqrt{1-y^2} - 1 + b\right)^2 dy = \underline{\underline{\text{für } b \leq 1!}}$$

$$\underline{\underline{V = \frac{1}{3b}\left(3(b-1)\arcsin\left(\sqrt{b(2-b)}\right) + \sqrt{b(2-b)}\cdot\left(b^2 - 2b + 3\right)\right)}} \checkmark$$

$h \neq R \neq 1$

$$V = \int_0^{\sqrt{1-(1-b)^2}} \frac{h}{b}\left(\sqrt{r^2-y^2} - r + b\right)^2 dy = \int_0^{\sqrt{1-(1-b)^2}} \frac{h\left(\sqrt{r^2-y^2} - r + b\right)^2}{b}\,dy$$

$$-\frac{\left(3hr^3 - 3bhr^2\right)\arcsin\left(\frac{\sqrt{2b-b^2}}{r}\right) + \sqrt{2b-b^2}\left((3hr - 3bh)\sqrt{r^2+b^2-2b} - 6hr^2 + 6bhr + \left(2b - 4b^2\right)h\right)}{3b}$$