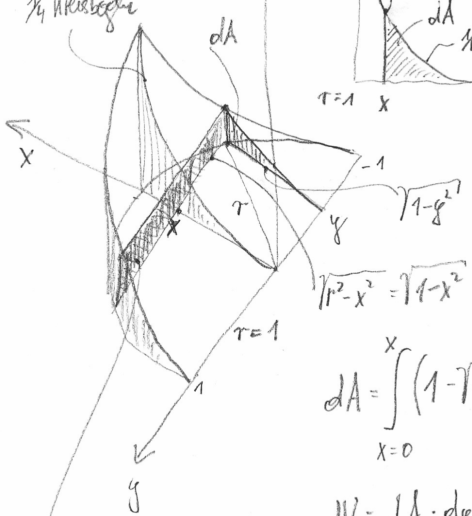
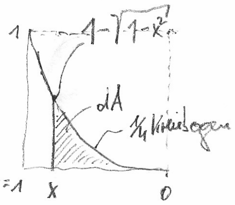
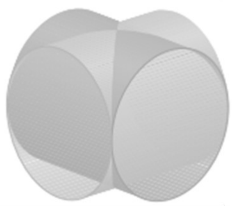
[Lit. 14]

Zylinderhuf mit zylindrischer „kreiskrummer" Dachfläche







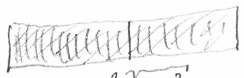
Vergl. Kreuzgewölbe,
Durchdringung zweier
(Halb-)Zylinder!

$$dA = \int_{x=0}^{x} \left(1-\sqrt{1-x^2}\right) dx$$

$$dV = dA \cdot dy$$

$$\underline{\underline{V}} = 2 \cdot \int_{y=0}^{1} dA\, dy = 2 \int_{y=0}^{1} \left(\int_{x=0}^{x=\sqrt{1-y^2}} \left(1-\sqrt{1-x^2}\right) dx \right) dy = \underline{\underline{\frac{\pi}{2} - \frac{4}{3} \approx 0{,}2374\ldots}}$$

Variante via „Rechtecksintegration"



$\}\, 1-\sqrt{1-x^2}$ bzw. $f(x) \to dA = 2\sqrt{1-x^2} \cdot f(x)$

$2 \cdot \sqrt{1-x^2}$

zB: $f(x) = x^2$

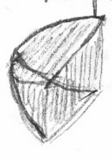
$dA = 2 \cdot \sqrt{1-x^2} \cdot x^2$

$V = \int_{x=0}^{1} 2 \cdot \sqrt{1-x^2} \cdot x^2\, dx = \pi/8$ ✓

$$dA = 2 \cdot \sqrt{1-x^2} \cdot \left(1-\sqrt{1-x^2}\right)$$
$$= 2 \left(\sqrt{1-x^2} - (1-x^2)\right)$$
$$= 2 \cdot \left(\sqrt{1-x^2} - 1 + x^2\right)$$

$$V = 2 \int_{x=0}^{1} \left(\sqrt{1-x^2} - 1 + x^2\right) dx = \underline{\underline{\frac{\pi}{2} - \frac{4}{3}}}$$

„Komplement"
$V_2 = 4/3$ $\Rightarrow$



Komplementkörper (Zylinderhuf)

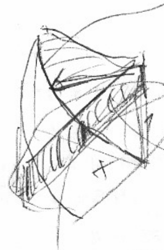
$$V_0 = V_{1/2\,Zylinder} = \frac{r^2\pi \cdot h}{2} = \pi/2$$
$(r = h = 1)$

$$V_1 = \frac{\pi}{2} - \frac{4}{3}$$

$$\int_{x=0}^{1} \left(\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1-x^2}\, dy \right) dx = 4/3$$

Komplementkörper „kopfstehend" aufgelegt!

Variante
Rechtecksintegration!



$f(x) = 1-\sqrt{1-x^2}$] 1/4 Kreisbogen

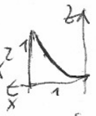
Rechteck! $\quad \overbrace{(1-\sqrt{1-x^2})}^{z = f(x)!}$

$2\cdot\sqrt{1-x^2}$ (immer!)

$2\cdot(\sqrt{1-x^2} - (1-x^2))$

$2\cdot(\sqrt{1-x^2} - 1 + x^2) = \frac{1}{6}(3\pi - 8) = 0{,}23746$

$\frac{\pi}{2} - \frac{4}{3}$ ✓ Ok ✓

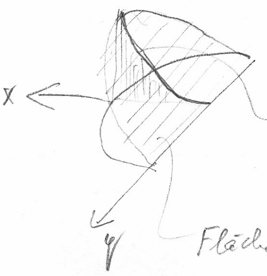


$$\int_{x=0}^{1} 2\cdot\sqrt{1-x^2}\cdot \boxed{x^2} \rightarrow \pi/8 ✓$$

$$\int_{x=0}^{1} \boxed{2\cdot\sqrt{1-x^2}}\cdot x^3 \rightarrow \frac{4}{15} ✓$$

usw.

Zylinderschnitt mit krummer Deckfläche. Krümmung in Abhängigkeit von einer Funktion



z.B.: $f(x) = z = x^2$

Fläche Geogebra: Wenn$(\sqrt{x^2+y^2} \le 1 \wedge x \ge 0, x^2)$ bzw. $f(x)$

$$V = \int_{x=0}^{1} \int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x)\,dy\,dx = \int_0^1 2 \cdot \int_0^{\sqrt{1-x^2}} f(x)\,dy\,dx$$

$f(x) = x^1 \longrightarrow V = 2/3$ ✓

$x^2 \longrightarrow \boxed{\pi/8}$ (komplizierte arcsin-Rechnung!) $\frac{\pi}{2^3}$ → mathdf.com

$x^3 \longrightarrow 4/15 = \frac{2^2}{3 \cdot 5}$

$x^4 \longrightarrow \boxed{\pi/16} = \frac{\pi}{2^4}$

$x^5 \longrightarrow \frac{16}{105} = \frac{4 \cdot 4}{15 \cdot 7} = \frac{2^4}{3 \cdot 5 \cdot 7}$

$x^6 \longrightarrow \frac{5\pi}{128} = \boxed{\frac{5\pi}{8 \cdot 16}} = \frac{5\pi}{2^7}$

$x^7 \longrightarrow \frac{32}{315} = \frac{4 \cdot 8}{15 \cdot 21} = \frac{2^5}{3 \cdot 3 \cdot 5 \cdot 7}$

$x^8 \longrightarrow \frac{7\pi}{256} = \frac{7\pi}{2^8}$

$x^9 \longrightarrow \frac{256}{3465} = \frac{2^8}{3 \cdot 3 \cdot 5 \cdot 7 \cdot 11}$

$x^{10} \longrightarrow \frac{21\pi}{1024} = \frac{3 \cdot 7\pi}{2^{10}}$