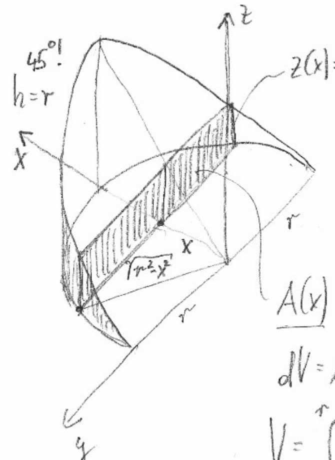
[Lit. 3]

Volumen eines Zylinderhufs: Integration via vertikale Rechteckschnitte

Volumen eines Zylinderhufs
Integration via vertikale Rechteckschnitte

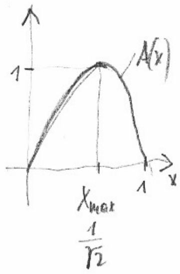


$$\underline{A(x)} = 2\cdot\sqrt{r^2-x^2}\cdot z = 2\sqrt{r^2-x^2}\cdot x$$

$$r=h=1 \qquad A(x) = 2\sqrt{1-x^2}\cdot x$$

$$dV = A(x)\cdot dx$$

$$\underline{\underline{V}} = \int_{x=0}^{r} 2\cdot\sqrt{r^2-x^2}\,x\,dx = -\frac{2}{3}\left(r^2-x^2\right)^{3/2}\Big|_0^r = \frac{2}{3}\left(r^2\right)^{3/2} = \underline{\underline{\frac{2}{3}r^3}}$$



$$r=h=1$$

$$\underline{\underline{V}} = \int_{x=0}^{1} 2\sqrt{1-x^2}\cdot x\,dx = \underline{\underline{2/3}}$$

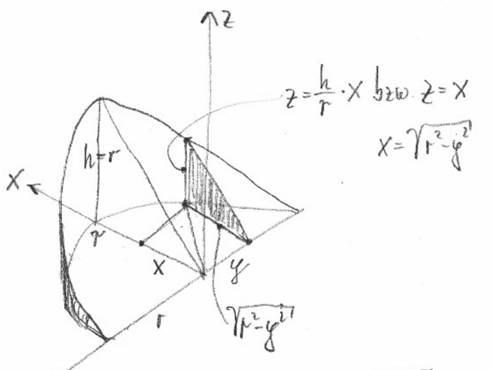
allgemein: $z = \frac{h}{r}\cdot x$

$$A(x) = 2\sqrt{r^2-x^2}\cdot\frac{h}{r}\cdot x$$

$$\underline{\underline{V}} = \int_{x=0}^{r} 2\sqrt{r^2-x^2}\cdot\frac{h}{r}\cdot x\,dx = \frac{2}{3}\sqrt{r^2-x^2}\cdot h\left(\frac{x^2}{r}-r\right)\Big|_0^r = \underline{\underline{\frac{2}{3}hr^2}}$$

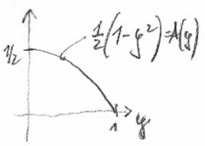
Volumen eines Zylinderhufs: Integration via vertikale Dreieckschnitte

Volumen eines Zylinderhufs
Integration via vertikale
Dreiecksschnitte



y

$$\underline{A(y)} = \frac{1}{2} \sqrt{r^2 - y^2} \cdot z = \frac{1}{2} \sqrt{r^2-y^2} \cdot \sqrt{r^2-y^2} = \underline{\frac{1}{2}\left(r^2 - y^2\right)}$$



$$\underline{\underline{V}} = 2 \int_{y=0}^{r} A(y) \cdot dy = 2 \int_{y=0}^{r} \frac{1}{2}\left(r^2 - y^2\right) dy = \int_{y=0}^{r} \left(r^2 - y^2\right) dy = \left(r^2 y - \frac{y^3}{3}\right)\Bigg|_0^r = \underline{\underline{\frac{2}{3} r^3}}$$

$r = h = 1$

$$\underline{\underline{V}} = \int_{y=0}^{1} \left(1 - y^2\right) dy = \underline{\underline{\frac{2}{3}}}$$

allgemein:

$$A(y) = \frac{1}{2} \sqrt{r^2 - y^2} \cdot \frac{h}{r} \cdot x = \frac{1}{2} \cdot \sqrt{r^2-y^2} \cdot \frac{h}{r} \sqrt{r^2-y^2} = \frac{1}{2} \cdot \frac{h}{r}\left(r^2 - y^2\right)$$

$$\underline{\underline{V}} = \int_{y=0}^{r} \frac{h}{r}\left(r^2 - y^2\right) dy = \left(hry - \frac{hy^3}{3r}\right)\Bigg|_0^r = \underline{\underline{\frac{2}{3} h r^2}}$$