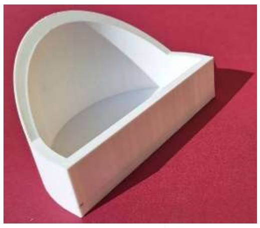
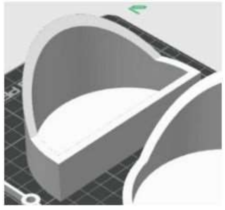
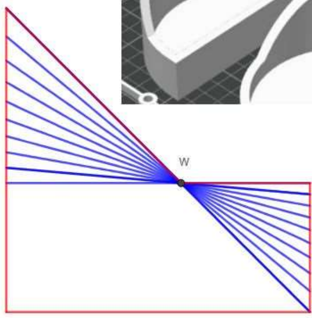
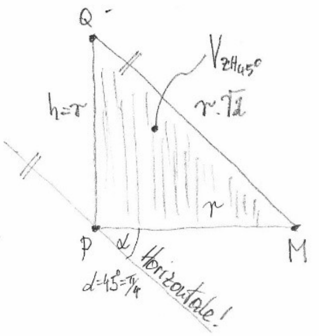
Wasserstand im speziellen Zylinderhuf-Halbzylinder-Modell





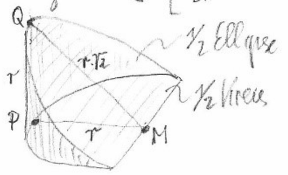


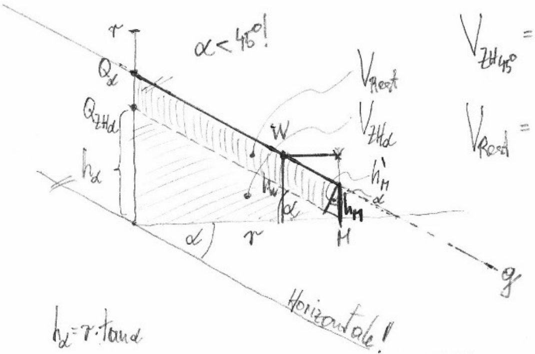


ZH = Zylinderhuf

[allgemein: $V_{ZH} = \frac{2}{3} r^2 \cdot h$]

$V_{45°\text{-}ZH} = \frac{2}{3} r^3$ [$h=r$]





$V_{ZH_{45°}} = V_{ZH_\alpha} + V_{Rest}$

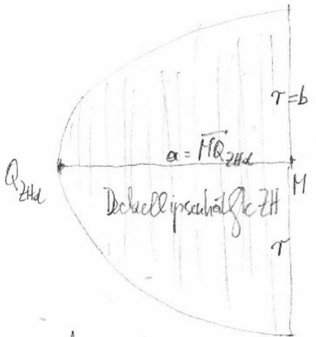
$V_{Rest} = V_{ZH_{45°}} - V_{ZH_\alpha} = \frac{2}{3} r^3 - \frac{2}{3} r^3 \tan\alpha$
$= \underbrace{\frac{2}{3} r^3 (1-\tan\alpha)}$

$\tan\alpha = h_\alpha / r$

$V_{ZH_\alpha} = \frac{2}{3} r^2 h_\alpha =$
$= \frac{2}{3} r^2 \cdot r \cdot \tan\alpha =$
$= \frac{2}{3} r^3 \tan\alpha$

$h_\alpha = r \cdot \tan\alpha$

$\overline{MQ}_{ZH_\alpha} = \sqrt{r^2 + r^2 \tan^2\alpha} = \sqrt{r^2(1+\tan^2\alpha)} =$
$= r\sqrt{1+\tan^2\alpha} = r \cdot 1/\cos\alpha$



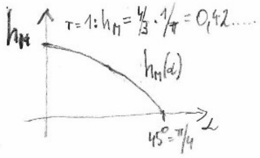
$A_{ell_{1/2}} = \frac{1}{2}$ „$a\,b\,\pi$" $= \frac{1}{2} r \frac{1}{\cos\alpha} r\pi = \frac{1}{2} r^2 \pi \frac{1}{\cos\alpha}$

$h'_M = h_M \cos\alpha$ $\quad V_{Rest} = A_{ell_{1/2}} h'_M = \frac{1}{2} r^2 \pi \cancel{\frac{1}{\cos\alpha}} \cdot h_M \cancel{\cos\alpha}$
$= \underbrace{\frac{1}{2} r^2 \pi h_M}$

$\frac{1}{2} r^2 \pi h_M = \frac{2}{3} r^3 (1-\tan\alpha)$

$$\boxed{h_M = \frac{4}{3} \cdot \frac{r}{\pi} (1-\tan\alpha)}$$

$\alpha = 45° = \pi/4$: $h_M = 0$ ✓
$\alpha = 0°$: $h_M = \frac{4}{3} \cdot \frac{r}{\pi}$ ✓



g: „$y = kx + d$"
$+x$ nach links!
$$\boxed{y = \tan\alpha \cdot x + \underbrace{\frac{4}{3} \cdot \frac{1}{\pi}(1-\tan\alpha)}_{h_M}}$$
$x = 0$: $y = h_M$
$x = \frac{4}{3}\frac{1}{\pi}$: $y = \frac{4}{3}\frac{1}{\pi}$

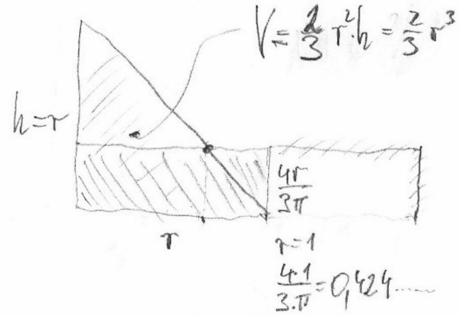
g: „$y = kx + d$" (x^{+} Richtung im Bild nach links!)

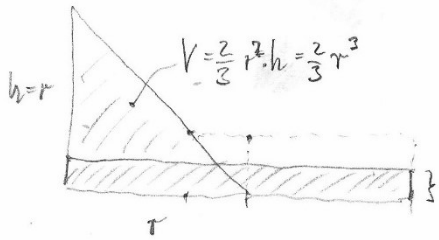
$$y = \tan\alpha \cdot x + h_M$$

$$y = \tan\alpha \cdot x + \frac{4r}{3\pi}(1-\tan\alpha) \qquad x=0:\ y = h_M \checkmark \qquad x=\frac{4r}{3\pi}:\ y=\frac{4r}{3\pi} \checkmark$$

$$\left[\begin{array}{l} x = \frac{4r}{3\pi} \\ y = \tan\alpha \cdot \frac{4r}{3\pi} + \frac{4r}{3\pi} - \frac{4r}{3\pi}\cdot\tan\alpha = \frac{4r}{3\pi} \end{array}\right. \qquad \underline{\underline{W\left(\frac{4r}{3\pi}\,\middle|\,\frac{4r}{3\pi}\right)}}$$

An der Stelle $x = \frac{4r}{3\pi}$ ist für jeden Neigungswinkel $0 \le \alpha \le 45°$ $y = \frac{4r}{3\pi}$!





$$\Big\}\ \frac{4r}{3\pi}\Big/2 = \frac{2r}{3\pi} \qquad \text{vergl.: } V_{Zylinder} = r^2\pi \cdot „h" = r^2\pi\left(\frac{2}{3}\frac{r}{\pi}\right) = \frac{2}{3}r^3 = \underline{\frac{2}{3}r^2\cdot h} \checkmark \quad (h=r)$$