MathDF

EN

Integral Calculator: Solve Definite and Indefinite Integrals (Antiderivatives)

Online integral calculator with step-by-step solutions, interactive graphs, and image scanning. Choose solution steps manually. Methods include u-substitution, integration by parts, partial fractions, trigonometric/hyperbolic/logarithmic formulas, polynomial long division, Euler substitution, binomial differentials, Weierstrass substitution, and Ostrogradsky's method. Solve definite and improper integrals using the Fundamental Theorem of Calculus.

INTEGRATION VARIABLE v

UPPER LIMIT r_t+r_k +∞

LOWER LIMIT r_t-r_k -∞

Print Screen & CTRL+V

sqrt(r_k^2-(v-r_t)^2)v

=

$$\int_{r_t - r_k}^{r_t + r_k} \sqrt{r_k^2 - (v - r_t)^2}\, v \, \mathrm{d}v$$

▸ Show input functions

Copy solution link

75% 90% 100% 110% 125%

Original integral

$$\int\limits_{r_t-r_k}^{r_t+r_k} v\sqrt{{r_k}^2-(v-r_t)^2}\,\mathrm{d}v$$

Value of the integral

$$\frac{\pi\,{r_k}^2\,r_t}{2}$$

Subject to the constraints

$$r_k>0,\quad r_k\neq 0$$

Step-by-step solution

Calculating

$$\int\limits_{r_t-r_k}^{r_t+r_k} v\sqrt{{r_k}^2-(v-r_t)^2}\,\mathrm{d}v$$

If the integrand $f(v)$ is continuous on the interval, then by the Fundamental Theorem of Calculus

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$$\int_a^b f(v)\,\mathrm{d}v = F(v)\Big|_a^b = F(b) - F(a)$$

where

$f(v)$ is continuous on the interval $[a,\ b]$

$F(v)$ is an antiderivative of $f(v)$

$$F(v)\Big|_{r_t - r_k}^{r_t + r_k} = F(r_t + r_k) - F(r_t - r_k)$$

Antiderivative

$$F(v) = \frac{{r_k}^2\, r_t\, \arcsin\left(\frac{v - r_t}{r_k}\right)}{2} + \sqrt{{r_k}^2 - (v - r_t)^2}\left(\frac{(v - r_t)^2}{3} + \frac{r_t\,(v - r_t)}{2} -\right.$$

Values at bounds:

$$F(r_t + r_k) = \frac{\pi\, {r_k}^2\, r_t}{4}$$

$$F(r_t - r_k) = -\frac{\pi\, {r_k}^2\, r_t}{4}$$

$$\frac{\pi\, {r_k}^2\, r_t}{2}$$

Verify by differentiating the antiderivative

Antiderivative

$$F(v) = \int v\,\sqrt{{r_k}^2 - (v - r_t)^2}\,\mathrm{d}v$$

$$= \frac{{r_k}^2\, r_t\, \arcsin\left(\frac{v - r_t}{r_k}\right)}{2} + \sqrt{{r_k}^2 - (v - r_t)^2}\left(\frac{(v - r_t)^2}{3} + \frac{r_t\,(v - r_t)}{2} - \frac{{r_k}^2}{3}\right) + C$$

Step-by-step solution

Calculating

$$\int v \sqrt{{r_k}^2 - (v - r_t)^2}\, \mathrm{d}v$$

Rewriting

$$\int v \sqrt{-v^2 + 2\, r_t\, v - {r_t}^2 + {r_k}^2}\, \mathrm{d}v$$

Substitution

$$x = v - r_t \quad \Bigg| \quad v = x + r_t, \quad \mathrm{d}v = \mathrm{d}x$$

[1]

$$\int -\frac{\sqrt{-r_t\,(-2\,x - 2\,r_t) - \frac{(-2\,x - 2\,r_t)^2}{4} - {r_t}^2 + {r_k}^2}\,(-2\,x - 2\,r_t)}{2}\, \mathrm{d}x$$

Substitution

$$u = \arcsin\left(\frac{x}{r_k}\right), \quad {r_k}^2 \cos^2(u) = {r_k}^2 - x^2 \quad \Bigg| \quad x = r_k \sin(u), \quad \mathrm{d}x = r_k \cos(u)\, \mathrm{d}u$$

[2]

$$\int -\frac{r_k \cos(u)\,(-2\,r_k \sin(u) - 2\,r_t)\sqrt{-\frac{(-2\,r_k \sin(u) - 2\,r_t)^2}{4} - r_t\,(-2\,r_k\, \mathrm{si}$$

$$2$$

Substitution

$$t = \sin(u), \quad \mathrm{d}t = \cos(u)\, \mathrm{d}u$$

[3]

$$-\frac{r_k}{2}\int (-2\,r_k\,t - 2\,r_t)\sqrt{-\frac{(-2\,r_k\,t - 2\,r_t)^2}{4} - r_t\,(-2\,r_k\,t - 2\,r_t) - {r_t}^2 + {r_k}^2}\, \mathrm{d}t$$

Substitution

$$z = -2\,r_k\,t - 2\,r_t \quad \left| \quad \begin{aligned} t &= -\frac{z + 2\,r_t}{2\,r_k} \\ \mathrm{d}t &= -\frac{1}{2\,r_k}\,\mathrm{d}z \end{aligned} \right.$$

[4]

$$-\frac{r_k}{2}\int -\frac{z\sqrt{-\frac{z^2}{4} - r_t\,z - {r_t}^2 + {r_k}^2}}{2\,r_k}\,\mathrm{d}z$$

Completing the square

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$$\frac{1}{4}\int \sqrt{{r_k}^2 - \left(\frac{z}{2} + r_t\right)^2}\,z\,\mathrm{d}z$$

Substitution

$$k = \frac{z}{2} + r_t \quad \left| \quad \begin{aligned} z &= 2\,k - 2\,r_t \\ \mathrm{d}z &= 2\,\mathrm{d}k \end{aligned} \right.$$

[5]

$$\frac{1}{4}\int 4\,(k - r_t)\,\sqrt{{r_k}^2 - k^2}\,\mathrm{d}k$$

Expand the parentheses

$$\int k\sqrt{{r_k}^2 - k^2} - r_t\sqrt{{r_k}^2 - k^2}\,\mathrm{d}k$$

Linearity

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$$\int k\sqrt{{r_k}^2 - k^2}\,\mathrm{d}k - r_t\int \sqrt{{r_k}^2 - k^2}\,\mathrm{d}k =$$

$$= (1) - r_t \cdot (2)$$

$$= \sqrt{{r_k}^2 - k^2}\left(\frac{k^2}{3} - \frac{r_t\,k}{2} - \frac{{r_k}^2}{3}\right) - \frac{{r_k}^2\,r_t\,\arcsin\left(\frac{k}{r_k}\right)}{2}$$

Back-substitution

$k = \frac{z}{2} + r_t \mid z = 2k - 2r_t$ [5]

$$\sqrt{{r_k}^2 - \left(\frac{z}{2} + r_t\right)^2}\left(-\frac{r_t\left(\frac{z}{2} + r_t\right)}{2} + \frac{\left(\frac{z}{2} + r_t\right)^2}{3} - \frac{{r_k}^2}{3}\right) - \frac{{r_k}^2 r_t \arcsin\left(\frac{\frac{z}{2} + r}{r_k}\right.}{2}$$

Back-substitution

$z = -2r_k t - 2r_t \mid t = -\frac{z + 2r_t}{2r_k}$ [4]

$$\frac{{r_k}^2 r_t \arcsin(t)}{2} + {r_k}^2\sqrt{1 - t^2}\left(\frac{r_k t^2}{3} + \frac{r_t t}{2} - \frac{r_k}{3}\right)$$

Back-substitution

$t = \sin(u)$ [3]

$$\frac{{r_k}^3 \sin^2(u)\sqrt{1 - \sin^2(u)}}{3} + \frac{{r_k}^2 r_t \sin(u)\sqrt{1 - \sin^2(u)}}{2} - \frac{{r_k}^3\sqrt{1 - \sin^2(u)}}{3}$$

Back-substitution

$u = \arcsin\left(\frac{x}{r_k}\right) \mid x = r_k \sin(u)$ [2]

$$\frac{{r_k}^2 r_t \arcsin\left(\frac{x}{r_k}\right)}{2} + \sqrt{{r_k}^2 - x^2}\left(\frac{x^2}{3} + \frac{r_t x}{2} - \frac{{r_k}^2}{3}\right)$$

Back-substitution

$x = v - r_t \mid v = x + r_t$ [1]

$$\frac{{r_k}^2 r_t \arcsin\left(\frac{v - r_t}{r_k}\right)}{2} + \sqrt{{r_k}^2 - (v - r_t)^2}\left(\frac{(v - r_t)^2}{3} + \frac{r_t(v - r_t)}{2} - \frac{{r_k}^2}{3}\right)$$

Calculating

$$\int k\,\sqrt{{r_k}^2-k^2}\;\mathrm{d}k \quad (1)$$

Substitution

$$p={r_k}^2-k^2$$
$$-\frac{1}{2}\,\mathrm{d}p=k\,\mathrm{d}k \quad [6]$$

$$\int -\frac{\sqrt{p}}{2}\;\mathrm{d}p$$

Power rule for integration

$$\int p^{\mathrm{n}}\;\mathrm{d}p=\frac{p^{\mathrm{n}+1}}{\mathrm{n}+1},\;(\mathrm{n}\neq-1)$$

where

$$\mathrm{n}=\frac{1}{2}$$

$$-\frac{1}{2}\cdot\frac{2\,p^{\frac{3}{2}}}{3}=-\frac{p^{\frac{3}{2}}}{3}$$

Back-substitution

$$p={r_k}^2-k^2 \quad [6]$$

$$\frac{\sqrt{{r_k}^2-k^2}\,\left(k^2-{r_k}^2\right)}{3}$$

Calculating

$$\int\sqrt{{r_k}^2-k^2}\;\mathrm{d}k \quad (2)$$

Substitution

$$p=\arcsin\left(\frac{k}{r_k}\right) \quad\Bigg|\quad k=r_k\,\sin\left(p\right)$$
$${r_k}^2\,\cos^2\left(p\right)={r_k}^2-k^2 \quad\Bigg|\quad \mathrm{d}k=r_k\,\cos\left(p\right)\,\mathrm{d}p \quad [7]$$

$$\int {r_k}^2 \cos^2(p)\,\mathrm{d}p$$

Applying the power-reduction formula

$$\cos^2(p) = \frac{1+\cos(2p)}{2}$$

$${r_k}^2 \int \frac{\cos(2p)+1}{2}\,\mathrm{d}p$$

Linearity

$${r_k}^2\left(\frac{1}{2}\int \cos(2p)\,\mathrm{d}p + \frac{1}{2}\int 1\,\mathrm{d}p\right) =$$

$$= {r_k}^2\left(\frac{1}{2}\cdot(3)_{\downarrow} + \frac{1}{2}\cdot(4)_{\downarrow}\right)$$

$$= \frac{{r_k}^2 \sin(2p)}{4} + \frac{{r_k}^2 p}{2}$$

Back-substitution

$$p = \arcsin\left(\frac{k}{r_k}\right) \;\Big|\; k = r_k \sin(p) \qquad [7]^{\uparrow}$$

$$\frac{{r_k}^2 \arcsin\left(\frac{k}{r_k}\right)}{2} + \frac{k\sqrt{{r_k}^2 - k^2}}{2}$$

Calculating

$$\int \cos(2p)\,\mathrm{d}p \qquad (3)^{\uparrow}$$

Substitution

$$q = 2p \;\Big|\; p = \frac{q}{2},\quad \mathrm{d}p = \frac{1}{2}\,\mathrm{d}q \qquad [8]_{\downarrow}$$

$$\int \frac{\cos(q)}{2}\,\mathrm{d}q$$

Standard integral

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$$\frac{1}{2}\cdot\sin(q) = \frac{\sin(q)}{2}$$

Back-substitution

$$q = 2\,p \;\Big|\; p = \frac{q}{2} \qquad [8]^{\uparrow}$$

$$\frac{\sin(2\,p)}{2}$$

Calculating

$$\int 1\,\mathrm{d}p \qquad (4)^{\uparrow}$$

Integral of a constant

$$p$$

$\sin(x)$ $\cos(x)$ $\arcsin(x)$ $\mathrm{d}x$ C

C = 0

r_k = 1

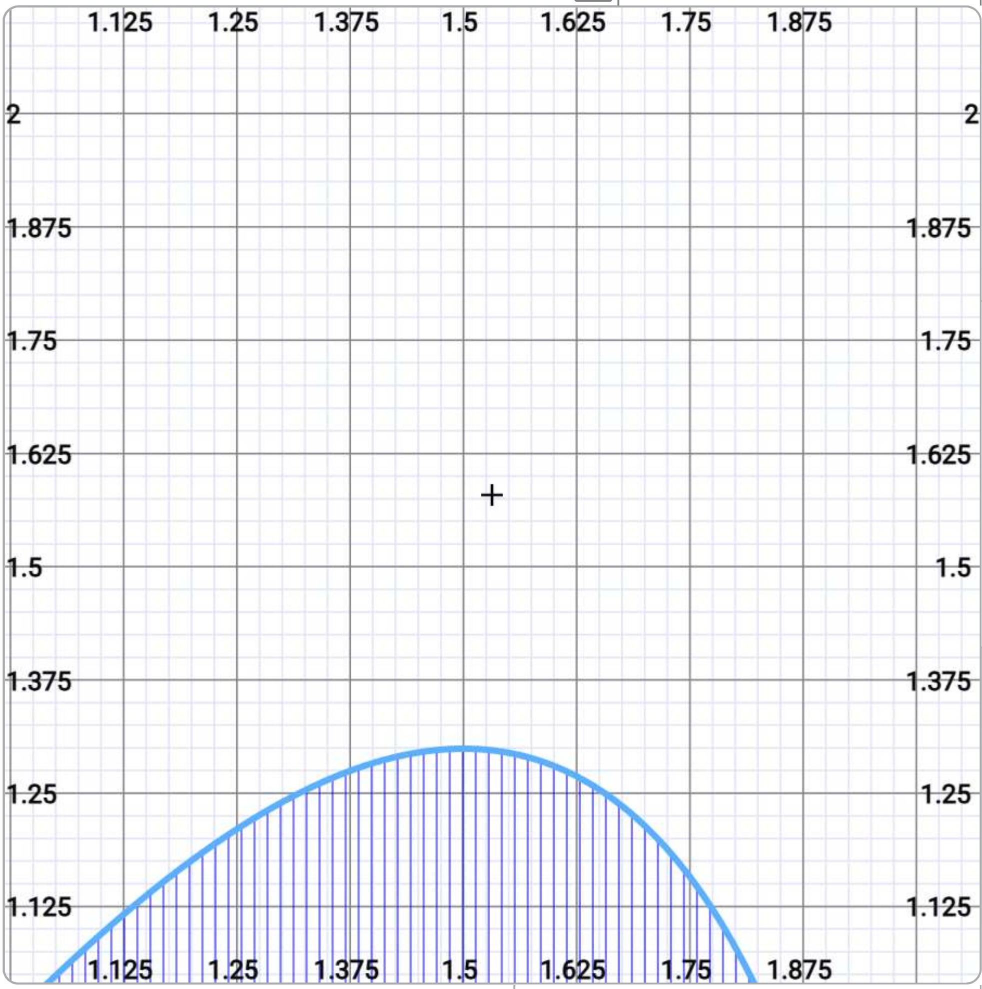
r_t = 1

Plot:

$f(x)$

$F(x)$

$$\int f(x)\,\mathrm{d}x = F(x)$$



x=1.5315, y=1.5791

Values:

$x =$

$f(x) =$ 1.297266

$F(x) =$ 0.302744

Zoom (wheel or gesture):

X : Y

X

Y

1 : 1

hello@mathdf.com