

Si Ci-Spirale: Krümmung bzw. Krümmungsradius

$$X(u) = a \cdot Ci(u) = -a \int \frac{\cos(t)}{t} dt$$

$$Y(u) = a \cdot Si(u) = a \int \frac{\sin(t)}{t} dt$$

Krümmung (Kappa) $\kappa = \frac{\dot{X}(u) \cdot \ddot{Y}(u) - \ddot{X}(u) \cdot \dot{Y}(u)}{(\dot{X}(u)^2 + \dot{Y}(u)^2)^{3/2}}$ Krümmungsradius (Rho) $\rho = \left| \frac{1}{\kappa} \right|$

$$\dot{X}(u) = a \cdot \frac{\cos(u)}{u}$$

$$\ddot{X} = a \cdot \frac{-\sin(u) \cdot u - \cos(u) \cdot 1}{u^2}$$

$$\dot{Y}(u) = a \cdot \frac{\sin(u)}{u}$$

$$\ddot{Y} = a \cdot \frac{\cos(u) \cdot u - \sin(u) \cdot 1}{u^2}$$

$$\kappa = \frac{\frac{a^2}{u^3} (\overbrace{\cos^2(u) \cdot u - \cos(u) \cdot \sin(u)}^{1 \cdot u} + \overbrace{\sin^2(u) \cdot u + \sin(u) \cdot \cos(u)})}{\left(a^2 \frac{\cos^2(u)}{u^2} + a^2 \frac{\sin^2(u)}{u^2} \right)^{3/2}}$$

$$= \frac{a \cdot \frac{\cos(u)}{u} \cdot a \cdot \frac{\cos(u) \cdot u - \sin(u)}{u^2} - a \cdot \frac{\sin(u)}{u} \cdot a \cdot \frac{-\sin(u) \cdot u - \cos(u)}{u^2}}{\left(\frac{a^2}{u^2} \cdot (\cos^2(u) + \sin^2(u)) \right)^{3/2}}$$

$$\sqrt[2]{\left(\frac{a^2}{u^2} \right)^3} = \left(\frac{a}{u} \right)^3$$

$$\kappa = \frac{\frac{a^2}{u^3} \cdot u}{\frac{a^3}{u^3}} = \frac{u}{a}$$

$$\kappa = \frac{u}{a} \quad \rho = \left| \frac{1}{\kappa} \right| = \left| \frac{a}{u} \right|$$

für $a=1$

$$\kappa = u \quad \rho = \left| \frac{1}{u} \right|$$