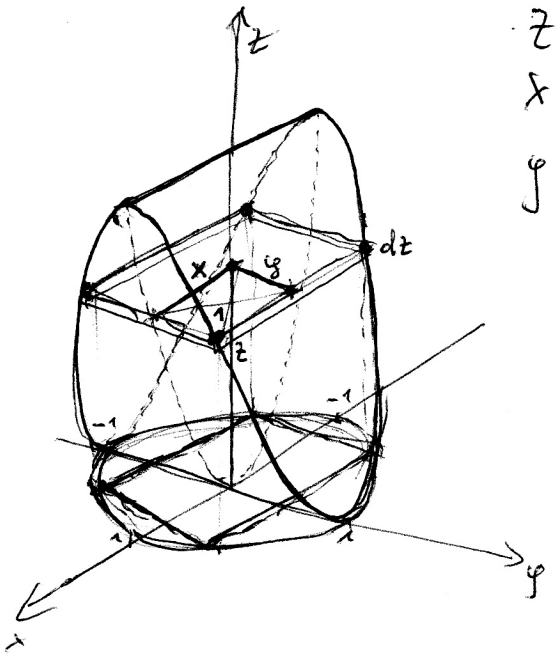
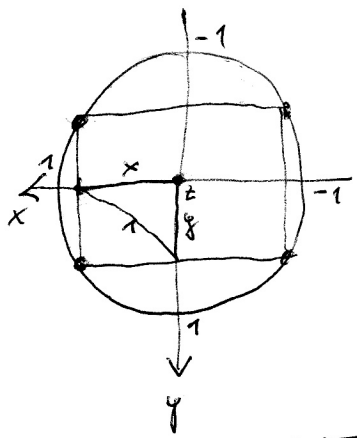
Volumen Rotationskörper via Rechteckschnitte

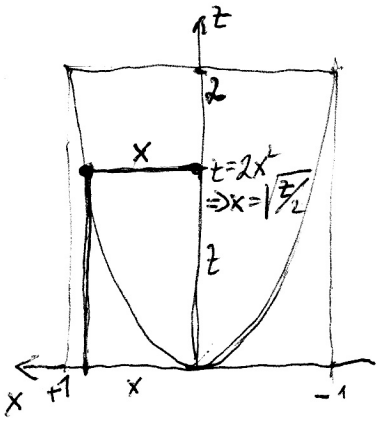


$z = 2x^2$

$x = \sqrt{\frac{z}{2}}$

$\varphi = \sqrt{1-x^2} = \sqrt{1-\frac{z}{2}} = \sqrt{\frac{2-z}{2}}$





$\underline{A(z)} = 4 \cdot x \cdot \varphi = 4 \cdot \sqrt{\frac{z}{2}} \cdot \sqrt{\frac{2-z}{2}} =$

$= \underline{2 \cdot \sqrt{z} \cdot \sqrt{2-z}} = \underline{2\sqrt{z(2-z)}}$

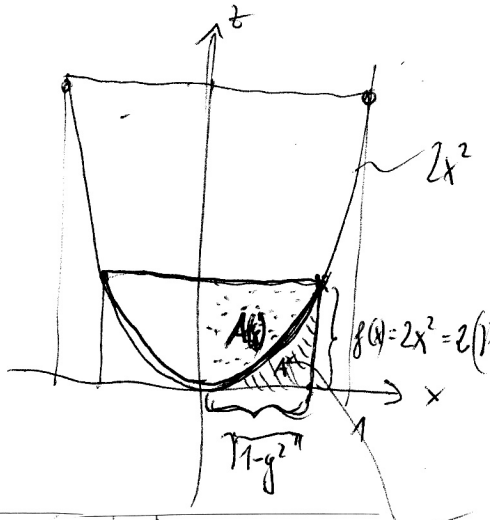
$\Rightarrow V = \int_0^2 2 \cdot \sqrt{z} \cdot \sqrt{2-z}\, dz = \pi$

z.B.: $x = 0{,}8 = 4/5$

$\varphi = \sqrt{1-x^2} = \sqrt{1-0{,}64} = \sqrt{0{,}36} = 0{,}6 = 3/5$

Schnitte nach Parabelsegmenten

Volumen 2Lagen via

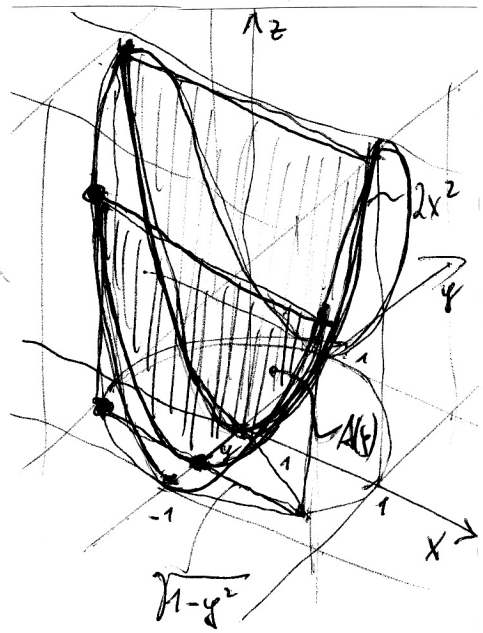


$A(y)$

$g(y) = 2x^2 = 2\left(\sqrt{1-y^2}\right)^2 = 2(1-y^2)$

$$A^*(y) = 2\int_0^{\sqrt{1-y^2}} g(x)\,dx \qquad 2\int_{x=0}^{x=\sqrt{1-y^2}} x^2\,dx = \frac{2}{3}x^3\Big|_0^{\sqrt{1-y^2}} = \frac{2}{3}\underbrace{\left(\sqrt{1-y^2}\right)^3}_{(1-y^2)\cdot\sqrt{1-y^2}}$$

$y=0: A^* = 2/3$ ✓
$y=1: A^* = 0$ ✓



$$A(y) = x\cdot g(x) - A^* = \underbrace{\sqrt{1-y^2}\cdot 2(1-y^2)}_{y=0\ \Delta=2,\ y=1\ A_\square=0} - \frac{2}{3}(1-y^2)\cdot\sqrt{1-y^2}$$

$$A(y) = \sqrt{1-y^2}(1-y^2)\cdot\left[2-\frac{2}{3}\right] = \frac{4}{3}\cdot(1-y^2)\cdot\sqrt{1-y^2}$$

$y=0: A(y) = 4/3$ ✓
$y=1: A(y) = 0$ ✓

x: 0 … 1 ↓ ×2 → x: −1 … 0 … 1 ↓
y: 0 … 1 ↓ ×2 → y: −1 … 0 … 1
} ×4 real

$$V = \frac{4}{2}\int_0^1 A(y)\,dy = 4\int_0^1 \frac{4}{3}(1-y^2)\cdot\sqrt{1-y^2}\,dy$$

$$= \frac{16}{3}\int_{y=0}^{y=1}(1-y^2)\cdot\sqrt{1-y^2}\,dy$$

$$= \frac{2}{3}\left(y\sqrt{1-y^2}\left(5-2y^2\right)+3\sin^{-1}(y)\right)\Big|_0^1$$

$\frac{2}{3}\cdot 3\cdot\sin^{-1}(1) = \frac{\pi}{2}$; 0

Kontrolle: (1. Ableitung)

$$\frac{16}{3}\left(1-y^2\right)^{3/2}$$ ✓

$$V = \pi$$