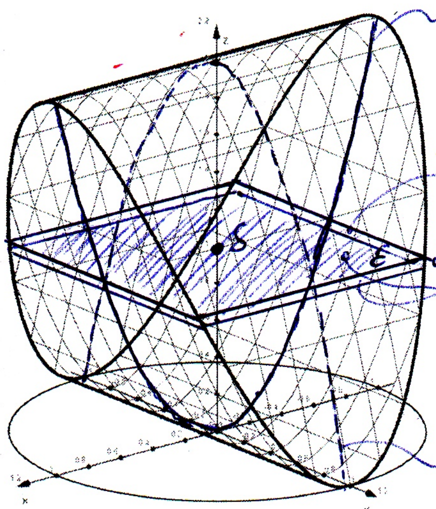
Zlapp Schwerpunkt



$z = 2x^2$

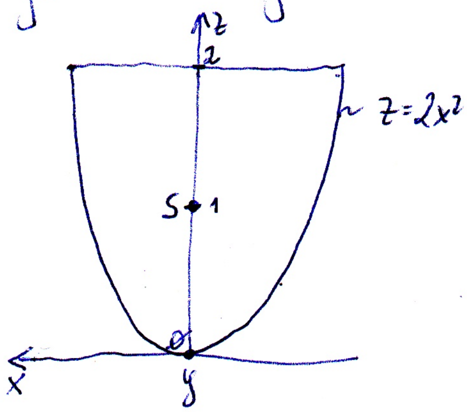
$a = 2y = 2\cdot\sqrt{\frac{2-z}{2}} = \sqrt{2}\cdot\sqrt{2-z}$

$b = 2x = 2\cdot\sqrt{\frac{z}{2}} = \sqrt{2}\cdot\sqrt{z}$

$dV = a\cdot b\cdot dz = 2\cdot\sqrt{z}\cdot\sqrt{2-z}\,dz$

$z = 2-2y^2$

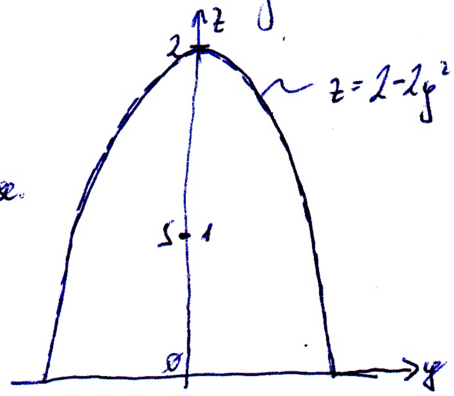
yz-Ebene als Symmetrieebene ⇒ S auf z-Achse



Plausibilitätsargumente vs. Formel!

ε, Ebene parallel zur xy-Ebene durch (0|0|1) teilt Zlapp in zwei massegleiche Körper.
Schwerpunkte der Teilkörper liegen gleich weit von der Teilungsebene ε entfernt, auf z-Achse.
S muss bez. der Schwerpunkte der Teilkörper in der Mitte liegen. ⇒ $S(0|0|1)$

xz-Ebene als Symmetrieebene ⇒ S auf z-Achse



via Formel:

$$z_S = \frac{\int_0^2 dm\cdot z = \int_0^2 \rho\, dV\cdot z = \rho\int_0^2 2\sqrt{z}\cdot\sqrt{2-z}\cdot z\,dz}{M_{ges} = \rho\cdot V_{ges} = \underbrace{\rho\cdot\int_0^2 2\sqrt{z}\,\sqrt{2-z}\,dz}_{\pi}} = \frac{\pi}{\pi} = 1 \checkmark$$

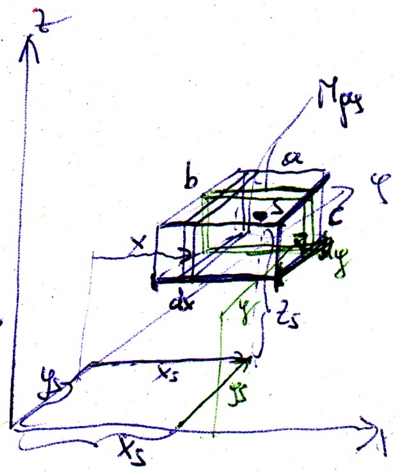
Schwerpunkt

$\rho = m/V$

$m = \rho \cdot V$

$dm = \rho \cdot dV$

$dV = dx \cdot dy \cdot dz$



$$x_s \cdot M_{ges} = \sum \Delta m_i \cdot x_i$$

$$= \int dm \cdot x$$

$$= \int \rho \cdot dV \cdot x = \rho \underbrace{\iiint}_{Bereich} x\, dx \cdot dy \cdot dz$$

$$M_{ges} = \rho \cdot V_{ges} = \rho \cdot \int dV = \rho \cdot \underbrace{\iiint}_{Bereich\ B} dx \cdot dy \cdot dz$$

$$\boxed{x_s = \frac{\rho \iiint_B x\, dx \cdot dy \cdot dz}{\underbrace{M_{ges}}_{= \rho \iiint_B dx\,dy\,dz}} = \frac{\iiint_B x\, dx\, dy\, dz}{\iiint_B dx\, dy\, dz}}$$

z.B.: Quader:

$\int dy \to b \quad \int dz \to c \quad \int dx = a$

$$x_s = \frac{b \cdot c \overbrace{\int x\, dx}^{a^2/2}}{b \cdot c \cdot a} = \frac{a \cdot b \cdot c \cdot a/2}{a \cdot b \cdot c} = a/2$$

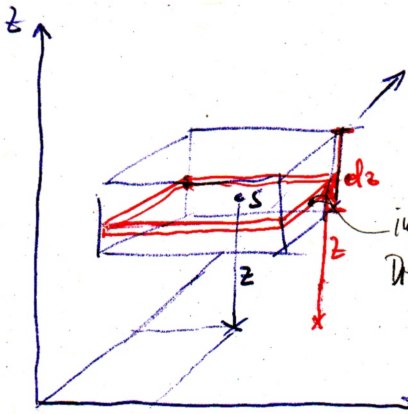
wenn Quader an yz-Ebene anliegt!

$$\boxed{y_s = \frac{\iiint_B y\, dx\, dy\, dz}{\iiint_B dx\, dy\, dz}}$$

$\int dx \to a$

$\int dz \to c$

Quader: $y_s = \frac{a \cdot c \int y\, dy}{a \cdot b \cdot c}$



infinitesimale Schicht dm

Drehmoment: $D = dm \cdot z$

$D = \int dm \cdot z$

$$\boxed{z_s = \frac{\iiint_B z\, dz\, dx\, dy}{\iiint_B dx\, dy\, dz}}$$

$\int dx \to a$

$\int dy \to b$

Quader: $z_s = \frac{a \cdot b \int z\, dz}{a \cdot b \cdot c}$