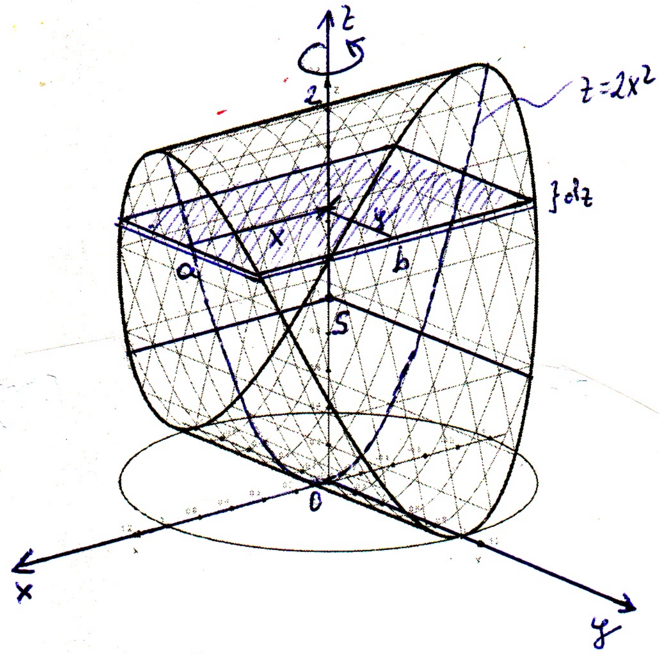


I_{ges}, z-Achse

$z = 2x^2$

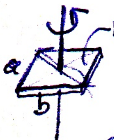
$x = \sqrt{z/2}$

$y = \sqrt{1-x^2} = \sqrt{1-\frac{z}{2}} = \sqrt{\frac{2-z}{2}}$

$a = 2y = 2\cdot\sqrt{\frac{2-z}{2}} = \sqrt{2}\cdot\sqrt{2-z}$

$b = 2x = 2\cdot\sqrt{z/2} = \sqrt{2}\cdot\sqrt{z}$

Trägheitsmoment Platte a mal b



$I = \frac{1}{12}\, m_{gesamt}\cdot(a^2+b^2)$

$\rho = m/V \rightarrow m = \rho\cdot V \rightarrow dm = \rho\cdot dV$ mit $\rho = 1$

$dm = a\cdot b\cdot dz$

$dI(z) = \frac{1}{12}\, dm\,(a^2+b^2) =$

$= \frac{1}{12}\, a\cdot b\,(a^2+b^2)\, dz$

$= \frac{1}{12}\cdot\underbrace{\sqrt{2}\cdot\sqrt{2-z}\cdot\sqrt{2}}_{2}\cdot\sqrt{z}\cdot\overbrace{(2\cdot(2-z)+2\cdot z)}^{2}\, dz$

$= \frac{\cancel{8}^{\,2}}{\cancel{12}_{\,3}}\sqrt{2-z}\cdot\sqrt{z}\cdot dz$

$$I_{ges.,z} = \int_{z=+0}^{z=+2} dI(z) = \frac{2}{3}\int_{z=+0}^{z=+2}\sqrt{2-z}\cdot\sqrt{z}\cdot dz = \frac{\pi}{3} = 1{,}047\ldots$$

n.b.

$$\int\sqrt{2-z}\cdot\sqrt{z}\cdot dz = \frac{1}{2}(z-1)\cdot\sqrt{(2-z)\cdot z} - \sin^{-1}\left(\sqrt{1-\frac{z}{2}}\right) + \text{const.}$$

$$\int_0^2\sqrt{2-z}\cdot\sqrt{z}\cdot dz = \frac{\pi}{2}$$

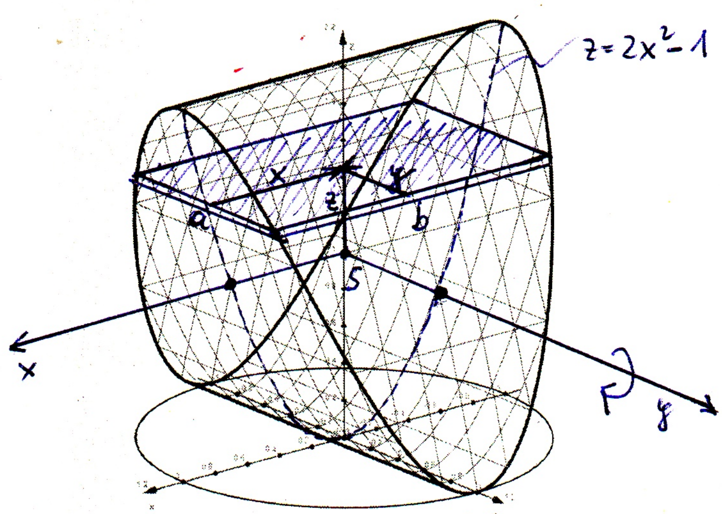
$\int_0^2$

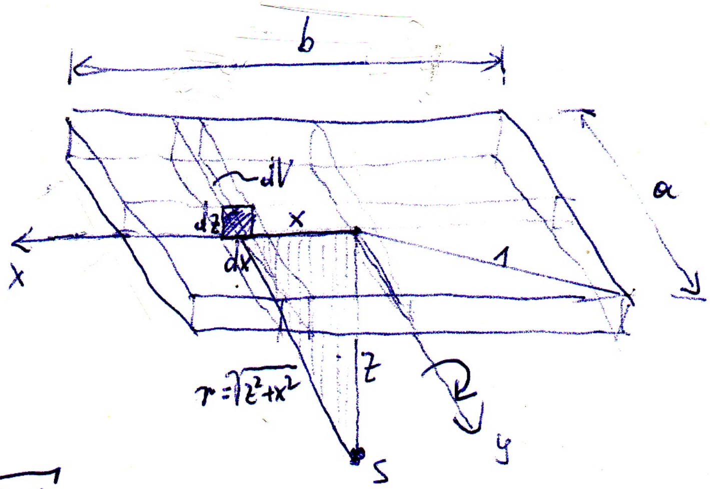
OG: $\frac{1}{2}\cdot 1\sqrt{0\cdot 2} - \sin^{-1}(\sqrt{0}) = 0 - 0 = 0$

UG: $\frac{1}{2}\cdot(-1)\sqrt{2\cdot 0} - \sin^{-1}(\sqrt{1}) = 0 - \frac{\pi}{2} = -\frac{\pi}{2}$

OG − UG $= 0 - (-\pi/2) = \pi/2$

I_{ges}, x- bzw y-Achse





$$z = 2x^2 - 1 \quad !$$

$$x = \sqrt{\frac{z+1}{2}} \qquad \underline{b} = 2x = 2\sqrt{\frac{z+1}{2}} = \underline{\sqrt{2}\cdot\sqrt{z+1}}$$

$$y = \sqrt{1-x^2} = \sqrt{1-\frac{z+1}{2}} = \sqrt{\frac{2-z-1}{2}} = \sqrt{\frac{1-z}{2}} \qquad \underline{a} = 2y = 2\cdot\sqrt{\frac{1-z}{2}} = \underline{\sqrt{2}\cdot\sqrt{1-z}}$$

$\rho = m/V \rightarrow m = \rho\cdot V$

$dm = \rho\cdot dV$ mit $\rho = 1 \rightarrow dm = dV$

$I = mr^2 \qquad dI = dm\,r^2 = \rho\,dV\,r^2 \rightarrow$ mit $\rho = 1 \rightarrow dV\,r^2$

$dV = dx\cdot dz\cdot a \qquad r = \sqrt{z^2+x^2}$

$dI = dV(z^2+x^2) = a\cdot(z^2+x^2)\cdot dx\cdot dz$

$$\frac{I_{ges,\ y\text{-Achse}}}{\text{bzw. } x\text{-Achse}} = \int_{z=-1}^{z=+1}\left[a\cdot\int_{x=-\sqrt{\frac{z+1}{2}}}^{x=+\sqrt{\frac{z+1}{2}}}(z^2+x^2)\,dx\right]dz = \int_{z=-1}^{z=+1}\left[a\left(z^2\int_{-\sqrt{\sim}}^{+\sqrt{\sim}}1\cdot dx + \int_{-\sqrt{\sim}}^{+\sqrt{\sim}}x^2\,dx\right)\right]dz = \int_{z=-1}^{z=+1}\left[a\left(z^2\cdot x\Big|_{-\sqrt{\sim}}^{+\sqrt{\sim}} + \frac{x^3}{3}\Big|_{-\sqrt{\sim}}^{+\sqrt{\sim}}\right)\right]dz =$$

$$= \int_{z=-1}^{z=+1}\left[a\left(z^2\underbrace{\left(\sqrt{\frac{z+1}{2}} - \left(-\sqrt{\frac{z+1}{2}}\right)\right)}_{2\cdot\sqrt{\frac{z+1}{2}} = \sqrt{2}\cdot\sqrt{z+1}} + \frac{1}{3}\underbrace{\left(\sqrt{\frac{z+1}{2}}^{\,3} - \left(-\sqrt{\frac{z+1}{2}}\right)^3\right)}_{\frac{2}{3}\cdot\sqrt{\frac{z+1}{2}}^{\,3} = \frac{2}{3}\cdot\frac{z+1}{2}\cdot\sqrt{\frac{z+1}{2}} = \frac{1}{3}\sqrt{2}\cdot\sqrt{z+1}\cdot\frac{z+1}{2}}\right)\right]dz =$$

$$a\sqrt{2}\cdot\sqrt{z+1}\left(z^2+\frac{1}{6}(z+1)\right) = a\cdot\sqrt{2}\cdot\sqrt{z+1}\cdot\frac{1}{6}(6z^2+z+1) \quad \text{mit } a = \sqrt{2}\cdot\sqrt{1-z}$$

$$\underbrace{\sqrt{2}\cdot\sqrt{1-z}\cdot\sqrt{2}}_{2}\cdot\sqrt{z+1}\cdot\frac{1}{\cancel{6}_{3}}(6z^2+z+1)$$

$$= \int_{z=-1}^{z=+1}\frac{1}{3}\underbrace{\sqrt{1-z}\cdot\sqrt{z+1}}_{=\sqrt{1-z^2}}\cdot(6z^2+z+1)\,dz = \frac{5\pi}{12} = 1{,}308\ldots$$

n.b. $\int\frac{1}{3}\sqrt{1-z}\cdot\sqrt{z+1}(6z^2+z+1)\,dz = \frac{1}{36}\left(\sqrt{1-z^2}(18z^3-4z^2-3z+4)+15\sin^{-1}(z)\right)+\text{const}$

OG: $\frac{1}{\cancel{36}_{3\cdot 12}}\left(0-\cancel{15}^{5}\frac{\pi}{2}\right) = \frac{5\pi}{24}$

UG: $-\frac{5\pi}{24}$

OG − UG $= \frac{5\pi}{12}$