# Drehmatrix

$$x' = x \cdot \cos\varphi - y \cdot \sin\varphi$$
$$y' = x \cdot \sin\varphi + y \cdot \cos\varphi$$

$\varphi = +45^\circ$ $\sin(45^\circ) = \cos(45^\circ) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \approx 0{,}71$

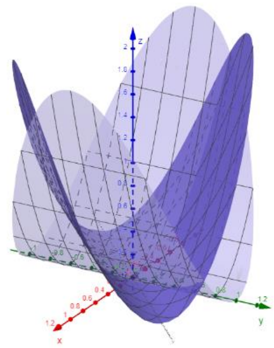
$$x' = \frac{\sqrt{2}}{2}(x-y)$$
$$y' = \frac{\sqrt{2}}{2}(x+y)$$

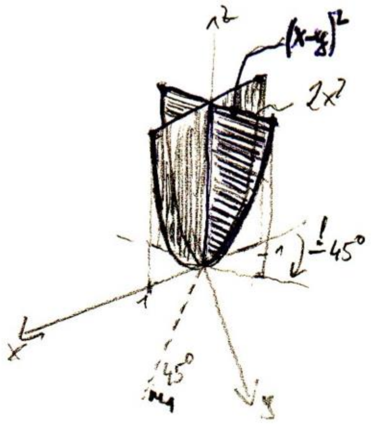
$\varphi = -45^\circ$ $\sin(-45^\circ) = -\frac{\sqrt{2}}{2}$ $\cos(-45^\circ) = \frac{\sqrt{2}}{2}$

$$x' = \frac{\sqrt{2}}{2}(x+y)$$
$$y' = \frac{\sqrt{2}}{2}(-x+y)$$

# Drehen der Funktionen

$$z = 2x^2$$
$$z = -2y^2+2$$






$z = 2x^2$

Transformationsgleichungen für $+45^\circ$! (*)

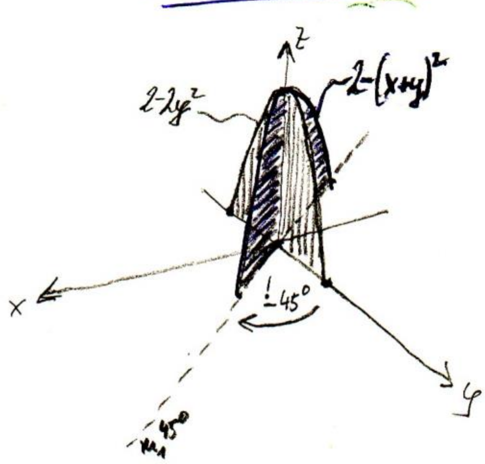
$x' = \frac{\sqrt{2}}{2}(x-y)$ $\quad z = 2(x')^2 = 2\cdot\left(\frac{\sqrt{2}}{2}\right)^2\cdot(x-y)^2 = \cancel{2}\cdot\frac{\cancel{2}}{\cancel{4}}(x-y)^2 = \underline{\underline{(x-y)^2 = z(x,y)}}$

$z = -2y^2+2$

Transformationsgleichungen für $+45^\circ$! (*)

$y' = \frac{\sqrt{2}}{2}(x+y)$ $\quad z = 2-2(y')^2 = 2 - 2\left(\frac{\sqrt{2}}{2}\right)^2\cdot(x+y)^2 = 2 - \cancel{2}\cdot\frac{\cancel{2}}{\cancel{4}}(x+y)^2 = \underline{\underline{2-(x+y)^2 = z(x,y)}}$

*) betreffend Transformation einer Funktion, siehe unten!



$z(x,y) = (x-y)^2 = x^2 - 2xy + y^2$

Variante zu Doppel-Volumen-Berechnung Kartesisch [Blatt 2a]

– 45° gedrehte Funktion $2x^2$

$$\int_{x=-1}^{x=+1}\int_{y=-\sqrt{1-x^2}}^{y=+\sqrt{1-x^2}} (x^2 - 2xy + y^2)\,dy\,dx =$$

3 Teile!

$$\int_{x=-1}^{1}\int_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}} x^2\,dy\,dx$$

$$\int_{x=-1}^{x=1} x^2 \left[\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\right] dx$$

$$\left[ y \right]_{y=-\sqrt{1-x^2}}^{y=+\sqrt{1-x^2}}$$

$$\overset{OG}{\sqrt{1-x^2}} - \left(-\overset{UG}{\sqrt{1-x^2}}\right) = 2\cdot\sqrt{1-x^2}$$

$$2\int_{x=-1}^{1} x^2 \cdot \sqrt{1-x^2}\,dx =$$

$$= 2\cdot\left[\frac{1}{8}\left(\underbrace{x\cdot\sqrt{1-x^2}\left(2x^2-1\right)}_{\text{immer } \emptyset \text{ wegen } \sqrt{1-x^2}\,!} + \sin^{-1}(x)\right)\right]_{x=-1}^{x=+1} =$$

$$= \cancel{2}\frac{1}{\cancel{8}_4}\left(\overset{OG}{\arcsin(1)} - \overset{UG}{\arcsin(-1)}\right) = \frac{\pi}{4}$$

$$\underbrace{\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)}_{\frac{2\pi}{2}}$$

$$\int_{x=-1}^{1}\left[\int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} y^2\,dy\right] dx$$

$$\left[\frac{y^3}{3}\right]_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}}$$

$$\frac{1}{3}\left(\sqrt{1-x^2}^3 - \left(-\sqrt{1-x^2}^3\right)\right)$$

$$\frac{2}{3}\sqrt{1-x^2}^3 = \frac{2}{3}\left(1-x^2\right)^{3/2}$$

$$\frac{2}{3}\int_{x=-1}^{1}\left(1-x^2\right)^{3/2} dx = \frac{2}{3}\cdot\frac{1}{8}\left[\underbrace{x\cdot\sqrt{1-x^2}\left(5-2x^2\right)}_{\text{immer } =\emptyset! \text{ wegen } \sqrt{1-x^2}} + 3\sin^{-1}(x)\right]_{-1}^{+1}$$

$$= \frac{1}{12}\left(\overset{OG}{3\cdot\arcsin(1)} - \overset{UG}{3\cdot\arcsin(-1)}\right)$$

$$= \frac{3}{12}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = \frac{1}{4}\cdot\frac{\cancel{2}\pi}{\cancel{2}} = \frac{\pi}{4}$$

$$\int_{x=-1}^{1}\int_{y=-\sqrt{1-x^2}}^{y=+\sqrt{1-x^2}} -2x\cdot y = -2\int_{x=-1}^{x=+1} x\left[\int_{y=-\sqrt{1-x^2}}^{y=+\sqrt{1-x^2}} y\,dy\right] dx =$$

$$\left[\frac{y^2}{2}\right]_{-\sqrt{1-x^2}}^{+\sqrt{1-x^2}}$$

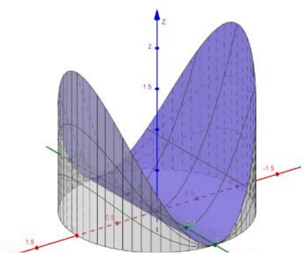
$$= -\cancel{2}\int_{x=-1}^{x=1} x\frac{1}{\cancel{2}}\left(\underbrace{\left(\overset{OG}{\sqrt{1-x^2}}\right)^2 - \left(-\sqrt{1-x^2}\right)^2}_{(1-x^2)\,-\,(1-x^2)}\right) dx =$$

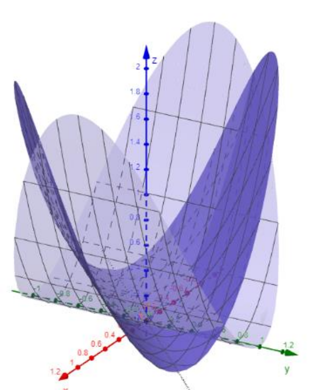
$$= -\int_{x=-1}^{x=1} x\cdot 0\cdot dx = \emptyset\,!$$

Begründung

oberhalb u. unterhalb der $xy$-Ebene liegende Flächen gegengleich groß $\Rightarrow \emptyset$

$$\Rightarrow \int_{x=-1}^{1}\int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \left(x^2 - 2xy - y^2\right) dy\,dx = \frac{\pi}{2}$$





Variante zu 2lapp-Volumen-Berechnung Blatt b)

−45° gedrehte Funktion $2-2y^2$

kartesisch

$$z(x,y) = 2-(x+y)^2 = 2-(x^2+2xy+y^2)$$

$$\int_{x=-1}^{x=+1}\left[\int_{y=-\sqrt{1-x^2}}^{y=+\sqrt{1-x^2}} 2-(x^2+2xy+y^2)\,dy\right]dx = \int_{x=-1}^{x=+1}\left[\int_{y=-\sqrt{1-x^2}}^{y=+\sqrt{1-x^2}} 2\,dy\right]\cdot dx - \underbrace{\int_{x=-1}^{x=+1}\left[\int_{y=-\sqrt{1-x^2}}^{y=+\sqrt{1-x^2}} (x^2+2xy+y^2)\cdot dy\right]dx}_{=\pi/2 \text{ vergl. Blatt a)!}} =$$

$$= 2\cdot\int_{x=-1}^{x=+1}\left[y\Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\right]\cdot dx = 2\cdot\int_{x=-1}^{x=+1}\underbrace{\left[\overset{OG}{\sqrt{1-x^2}}-\left(\overset{UG}{-\sqrt{1-x^2}}\right)\right]}_{2\cdot\sqrt{1-x^2}}dx =$$

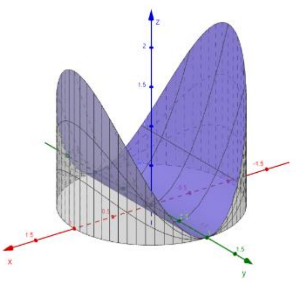
$$= 4\cdot\int_{x=-1}^{x=+1}\sqrt{1-x^2}\,dx = \overset{2}{\cancel{4}}\left(\frac{1}{2}\left(\underbrace{\sqrt{1-x^2}\cdot x}_{\substack{x=\pm1\to 0\\ \text{immer } 0}}+\sin^{-1}(x)\right)\right)\Bigg|_{-1}^{1} =$$

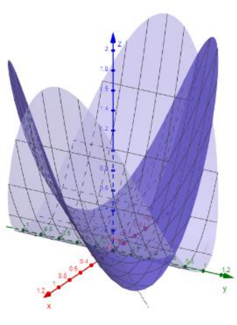
$$= 2\cdot\left(\underbrace{\underbrace{\sin^{-1}(1)}_{\frac{\pi}{2}}-\underbrace{\sin^{-1}(-1)}_{\left(-\frac{\pi}{2}\right)}}_{\pi}\right) = 2\pi$$

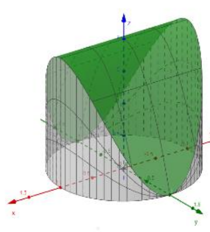
$$2\pi - \pi/2 = 3\pi/2$$

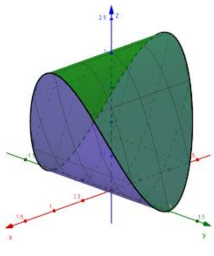
$$V_{2lapp} = \overset{\text{s. Blatt b)}}{\frac{3\pi}{2}} - \overset{\text{s. Blatt a)}}{\frac{\pi}{2}} = \pi$$

w.z.z.w.



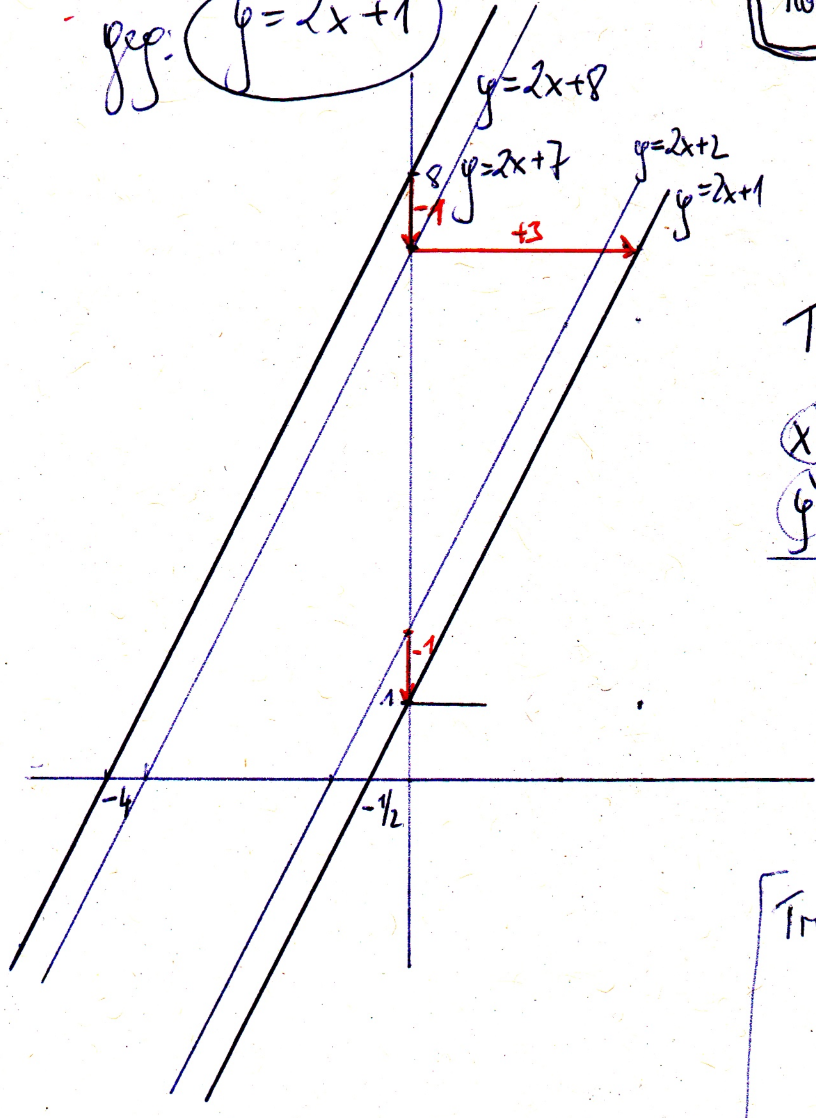






betreffend Transformation (Translation, Rotation) einer Funktion

geg: $y = 2x + 1$




Translation:

$$x' = x + 3$$
$$y' = y - 1$$

$$y = 2x + 1$$
$$y - 1 = 2(x+3) + 1$$
$$y = 2x + 8$$

Translationsteil a)

$$x' = x + 3$$
$$y' = y$$

Verschiebung nur in x-Richtung

$$y = 2(x+3) + 1$$
$$y = 2x + 7$$

Transformationsteil b)

$$x' = x$$
$$y' = y - 1$$

Verschiebung nur in y-Richtung

$$y - 1 = 2x + 1$$
$$y = 2x + 2$$

notabene!

Durch Einsetzen der Transformationsterme ($x'=x+3$; $y'=y-1$) in die gegebene Gleichung ($y=2x+1$) erhält man eine Gleichung, die ein Objekt beschreibt, aus dem unter der gegebenen Transformation das gegebene Objekt ($y=2x+1$) erst entsteht. Mit anderen Worten: Die beschriebene Vorgangsweise liefert das Inverse zu der mit den Transformationstermen ausgedrückten Operation.

salopp: z.B.: statt einer Translation nach rechts, eine Translation nach links.
statt einer Drehung um $+\alpha^\circ$, eine Drehung um $-\alpha^\circ$